Experimental Investigation of the Expansion of Moist Air around a Sharp Corner

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The results of an experimental study of two-dimensional supersonic expansions of moist air around a sharp corner are reported. Condensation of water vapor by homogeneous nucleation occurs within the expansion fan. Measurements of the location of the onset of condensation show that the leading edge of the condensation zone is concave with respect to the oncoming flow. Density measurements, obtained from the analysis of flow-interferograms, indicate that the flowfield within the condensation zone is a function of both expansion angle and radial distance from the corner. An empirical relation between experimentally determined adiabatic supercooling and expansion cooling rate is obtained.

Nomenclature

C = constant, Eq. (5)
h^* = nozzle throat height
K = Gladstone-Dale constant
L = optical path length
M = Mach number
n = exponent, Eq. (5)
p = pressure
R = universal gas constant
r = radial distance measured from the corner
ΔS_{12} = nondimensional fringe shift
T = temperature
ΔT_{ad} = adiabatic supercooling
γ = ratio of specific heats
λ = wavelength
μ = molecular weight
ρ = density
Φ = relative humidity
ϕ = expansion angle measured from the vertical at the corner
ω = specific humidity

Subscripts

a = air
c = saturated conditions, condensed phase
k = onset of condensation
v = water vapor
0 = stagnation conditions
1 = reference conditions
2 = measured properties
∞ = plane surface

Introduction

SUPERSONIC expansions of moist air can lead to situations in which water vapor condenses at highly supersaturated conditions, i.e., where the relative humidity of the mixture is greater than 100%. Except in the presence of extreme gradients such as those produced by shock waves, velocity and temperature differences between the condensed particles and the gaseous part of the flow are negligible in expansions. The only readily measurable effect of condensation is that of heat addition to the flow of a mixture of thermally and calorically perfect gases.[1,2] This paper reports some experimental results of the study of two-dimensional expansions of moist air around a sharp corner, with condensation by homogeneous nucleation occurring within the expansion fan. Interferograms of the flow are analyzed to obtain information on the onset of condensation and on the shape of the condensation front.

Two-dimensional corner expansions with heat addition have been treated in two different ways. For one, Samaras[3] and Ross[4] computed the changes in flow properties across an oblique heating front using integral relations. To date, the only investigation to consider continuous heat addition appears to be that of Steffen,[5] who computed the velocity field by assuming that the flow through the heat addition zone remains a Prandtl-Meyer type expansion. The theory is verified qualitatively by experiments in which the heat addition is accomplished by the condensation of water vapor.

In applying Steffen's analysis, one is restricted to cases in which the amount of heat added to the flow is constant along radial lines from the corner, i.e., the amount of heat added does not depend on the residence time of the gas between two radial lines. However, heat addition resulting from condensation is a rate process. Here, the rate of heat addition is the product of the condensation rate and the latent heat of condensation. Since the residence time of the gas between two radial lines becomes increasingly larger as we proceed outward from the corner, we expect to find the amount of heat addition along a ray increasing with distance from the corner, and the flow within the condensation zone will not be of Prandtl-Meyer form.

Condensation that occurs in the absence of dust or other condensation sites is defined as homogeneous nucleation. Centers for the growth of the condensed phase are produced spontaneously in the vapor itself at supersaturated conditions, i.e., where $p_v/p_\infty > 1$.[6] Small, unstable clusters are first formed by fluctuations of the vapor molecules. Some of these grow to a critical size, at which the addition of one more molecule leads to sudden condensation and further cluster growth, which is then controlled by heat and mass transfer between the condensed and vapor phases.

A convenient measure of the onset of condensation is the adiabatic supercooling, ΔT_{ad}, defined by

$$\Delta T_{ad} = T_c - T_k \tag{1}$$

where T_k is the temperature at which condensation first occurs, and T_c is the temperature at which saturation with respect to a plane liquid or solid surface occurs. In nozzle expansions of moist air, supercooling is found to be directly proportional to the temperature gradient at the nozzle throat. For high relative humidity in the nozzle supply, supercoolings of from 50 to 100°C over a range of cooling rates of from 0.3 to 3°C/μsec have been reported.[7,8] Since the cooling rate in a Prandtl-Meyer expansion (neglecting viscous effects) varies from ∞ at the corner to nearly zero at large distances from the corner, we expect to find decreasing values of ΔT_{ad} as we

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proceed outward from the corner, resulting in a condensation front that is concave with respect to the oncoming flow.

Experiments

Experiments designed to investigate the aforementioned features of corner expansions with condensation were carried out in a Ludwieg tube intermittent supersonic wind tunnel, the design, instrumentation, and performance of which are given in Ref. 9. This facility produces steady flow of about 15 msec duration through a two-dimensional nozzle having a 2-in. × 2-in. sonic throat. The lower nozzle block terminated in a 40° sharp corner located at the geometric throat. Vertical fringe interferograms of the expansion were obtained with a Mach-Zehnder interferometer. A 1 μsec duration spark source served as a point light source, and monochromatic light was produced by a 5800 Å interference filter. The tunnel was supplied with moist room air, which was bubbled through water in order to raise its specific humidity. The supply relative humidity Φ_0 was set at the desired experimental value without changing the specific humidity by regulating the supply pressure. The stagnation properties at the nozzle entrance were computed from measured tunnel supply temperature, pressure, and relative humidity, using standard gasdynamic equations for Ludwieg tube operation.[9] Typical stagnation properties were $T_0 = 284°K$; $370 \text{ torr} \leq p_0 \leq 670$ torr; $40\% \leq \Phi_0 \leq 70\%$.

Analysis and Results

The basic quantity determined from the interferograms is the nondimensional fringe shift, ΔS_{12}, measured in fringe numbers from a region of known density ρ_1 to a region of unknown density ρ_2. For a two-dimensional flowfield, ρ_2 is determined from

$$\rho_2 = \rho_1 + (\lambda/K \cdot L) \cdot \Delta S_{12} \tag{2}$$

where K, the Gladstone-Dale constant, is given by

$$K = (1 - \omega) \cdot K_a + \omega \cdot K_v \tag{3}$$

for a mixture of perfect vapors. Here, ω is the specific humidity, and the Gladstone-Dale constants for air and water vapor are $K_a = 0.226$ cm³/g and $K_v = 0.316$ cm³/g, respectively, for a wavelength of 5893 Å.[10] Because of the low specific humidity encountered in the experiments, K is evaluated using the nozzle supply specific humidity ω_0 and is assumed to be constant throughout the expansion. Thus, we neglect the presence of condensed particles in determining K. Since the mass fraction of the condensate must be less than or equal to the small specific humidity in the supply ($\omega_0 \simeq 0.006$), the resulting error in the density measurements is negligible. All density measurements were made with respect to a vertical reference line established upstream from the corner. The reference density ρ_1 along this line was experimentally determined from horizontal fringe interferograms of dry air flow around the corner.

Measured densities for the corner expansion of dry air are shown in Fig. 1. The ordinate is made dimensionless by dividing the measured density ρ_2 by the stagnation density ρ_0 and the results are plotted against the expansion angle ϕ measured from the vertical at the corner. Within the experimental error of the measurements, points measured along four lines of constant radial distance from the corner form a single curve, which is taken to be the isentropic expansion. The radial distance r is made dimensionless by dividing by the nozzle throat height h^*. Here, the use of h^* as a characteristic length, rather than the product of a characteristic velocity and a characteristic nucleation time, is suggested by the physical boundaries of the problem. Thus, near the corner ($r/h^* < 0.2$), the boundary layer produces large deviations in measured density from the isentrope shown in Fig. 1,[11,12] and causes a departure from theoretical Prandtl-Meyer flow. For $r/h^* > 0.5$, waves reflected from the upper nozzle block interact with the expansion fan to produce a two-dimensional flowfield.

Figure 2 shows the measured density for the case of moist air expansion. The flow is isentropic up to the condensation zone, indicated by the region of increasing density (heat addition in supersonic flow drives the Mach number toward unity and causes pressure, temperature, and density to increase). The flow within the condensation is two dimensional, and, as can be seen from the difference between measured density and that of the isentropic expansion, heat addition increases with radial distance from the corner.

The onset of condensation was measured by two different methods. For one, an increase of 1% between the measured density and the isentropic flow density was used as a criterion for determining onset. The second method involved measuring changes in fringe curvature. The interferometer was adjusted, so that, in an isentropic expansion, fringes continuously curved to the left. Then, the points on the interferograms of flow with condensation, where fringes began curving to the right, signaling an increase in density, were taken to be the location of onset. The error in determining ϕ at onset was estimated to be ±2° for both methods. Experimentally determined onset points for cases with three different relative humidities at the nozzle entrance are shown in Fig. 3. Open symbols represent points obtained by measuring changes in fringe curvature, while the shaded circles of Experiment No. 29 represent points obtained from density measurements. In all cases, the condensation front is concave with respect to the

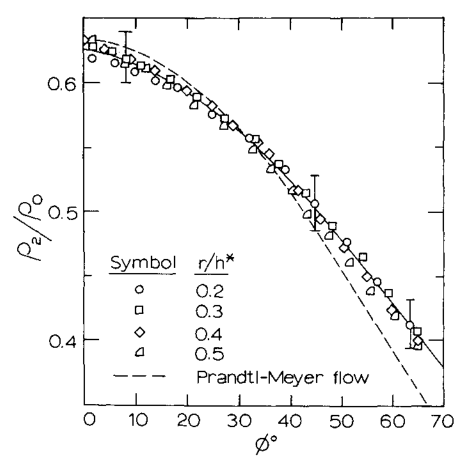


Fig. 1 Measured density for the corner expansion of dry air: experiment 37, T_0 = 284°K, p_0 = 578 torr, $\omega_0 \simeq 0.02$ g/kg.

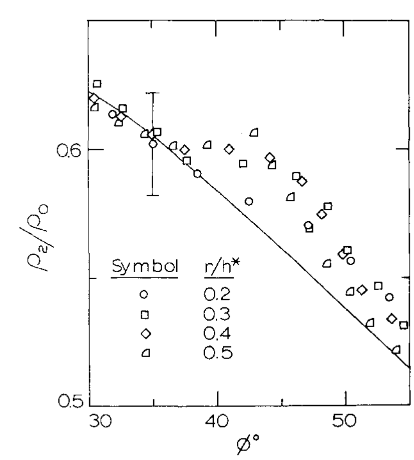


Fig. 2 Measured density for the corner expansion of moist air: experiment 29, T_0 = 284°K, p_0 = 577 torr, ω_0 = 5.8 g/kg, Φ_0 = 54%.

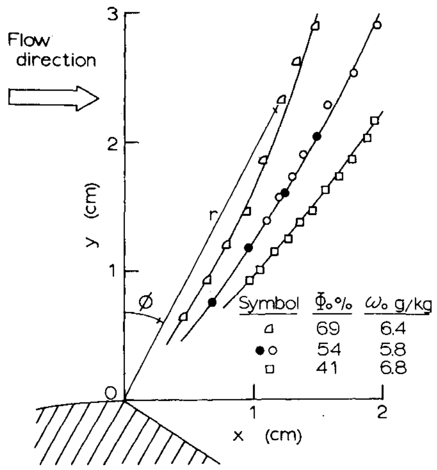


Fig. 3 Measured onset of condensation.

oncoming flow. As in the case of nozzle expansions, condensation occurs farther upstream as the relative humidity in the supply is increased.

In Fig. 4, the adiabatic supercooling, calculated at the measured onset points, is plotted against the expansion cooling rate dT/dt given by

$$\frac{dT}{dt} = -\frac{2(\gamma - 1)}{(\gamma + 1)} \cdot \left[\frac{R}{\mu} \cdot (M^2 - 1)\right]^{1/2} \cdot \frac{T^{3/2}}{r} \tag{4}$$

The error bars shown in Fig. 4 correspond to the estimated error of $\pm 2°$ in determining ϕ at onset; supercooling is seen to be relatively independent of Φ_0. We note that the measured supercooling is considerably below that reported for condensation in nozzle expansions at identical cooling rates.[7,8] There, however, the cooling rate is that computed at the nozzle throat, while in fact, condensation takes place downstream from the throat at a different cooling rate. Equation (4) gives the local value of the cooling rate, computed at the measured onset of condensation. In addition, since homogeneous nucleation results in the production of critical nuclei that are more numerous than nuclei (dust) in laboratory air by factors as high as 10^8 (Ref. 6), we may rule out dust in the tunnel supply acting as condensation sites, which would cause lower supercooling than in the case of condensation by pure homogeneous nucleation.

Also shown in Fig. 4 is the result of fitting the empirical equation

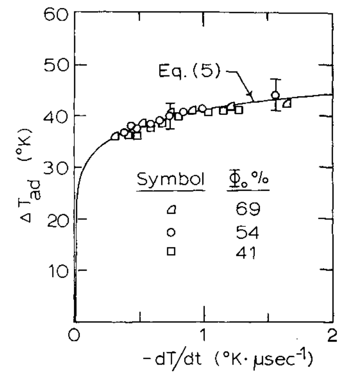
$$\Delta T_{ad} = C \cdot (dT/dt)^n \tag{5}$$

to the data; the constants in Eq. (5) are found to be C = 40.8 μsec and n = 0.121. The form of Eq. (5) is suggested by the values of ΔT_{ad} in the limits of frozen and equilibrium flow. Thus, immediately at the corner, where $dT/dt = \infty$, Eq. (5) gives the frozen flow condition of infinite supercooling. For $dT/dt = 0$, corresponding to an infinite distance from the corner, $\Delta T_{ad} = 0$, which is the proper limiting condition for equilibrium flow. With the local cooling rate expressed by Eq. (4), the adiabatic supercooling given in Eq. (5) is seen to be a function of the local Mach number, radial distance from the corner, and the local static temperature (or, since the flow is isentropic up to the onset of condensation, the stagnation temperature, T_0).

Conclusions

As a result of the present investigation of corner expansions with water vapor condensation occurring within the expansion fan, the following conclusions can be stated. 1) The expansion is isentropic up to the onset of condensation, while the flowfield within the condensation zone is, as expected, a function of both expansion angle and radial distance from the corner. 2) The condensation front is concave with respect to the oncoming flow. As the relative humidity in the supply is increased, condensation onset occurs farther upstream in the expansion. 3) For the range of relative humidities studied, the adiabatic supercooling is a function only of the local cooling rate in the expansion fan. An empirical relation between supercooling and cooling rate is seen to satisfy the limiting conditions of frozen flow at the corner and equilibrium flow far from the corner. 4) The findings indicate that the basic assumption of the analysis of Steffen, i.e., constant amount of heat addition along radial lines from the corner, is not applicable to corner expansions with water vapor condensation. Instead, it is suggested that the curved condensation front provides the proper boundary condition for a two-dimensional, theoretical treatment of the problem.

Fig. 4 Adiabatic supercooling computed at measured onset points.



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